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Lecture 5

$p \to q$
q
$\therefore q$

p	q	$p \to q$	$(p \to q) \wedge p$	$([p \to q]) \wedge p] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

- The process above is to time consuming so use the rules of inference.

Example of proving argument:

$\neg r$
$\neg p \to q$
$q \to r$
$\therefore p$

1. $\neg p \to q$ hypothesis
2. $q \to r$ hypothesis
3. $\neg p \to r$ hypothetical syllogism 1, 2
4. $\neg r$ hypothesis
5. p Modus tollens 3, 4

Rules of inference with quantifiers

Universal Generalization:

c is an arbitrary element
$P(c)$

$\therefore \forall x\, P(x)$

- Since $P(c)$ is true and c is ARBITRARY then $P(x)$ must be true for all elements in domain.

Example:

$\forall x\, [P(x) \vee B(x)]$
$\exists x\, [\neg P(x)]$

$\exists x\, B(x)$

1. $\exists x\, [\neg P(x)]$ hypothesis
2. c is a specific element $\wedge \neg P(c)$ existential instantiation
3. $\neg p(c)$ simplification 2
4. c is a specific element simplification 2
5. $\forall x\, [P(x) \vee B(x)]$ hypothesis
6. $P(c) \vee B(c)$ universal instantiation
7. $B(c)$ disjunctive syllogism 3, 6
8. $\exists x\, [B(x)]$ existential generalization 7

- A theorem is a statement that has been proved using:
    - definitions
    - other theorems
    - axioms (statements which are assumed to be true)

- Lemma → Theorem → Corollary